KBM Journal of Mathematical Sciences & Computer Applications (2010) 1 (1): 1–8
doi: 10.5147/kbmjmsca/2010/0004

Remarks on Closure and Interior Operators in Bitopological Spaces

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Received January 20, 2010 / Accepted April 30, 2010

Abstract

Andrijevic and his collaborators studied the various aspects of closure and interior operators in general topological spaces and obtained several relations among them. Researchers in general topology studied such operators in bitopological settings. Andrijevic established that the result $\alpha cl A = A \cup cl(int(cl A))$ holds for any subset A of a topological space where $\alpha cl A$, $cl A$ and $int A$ denote the α-closure of A, closure of A and interior of A respectively. He also established the analog results for other operators in terms of the closure and interior operators in general topological spaces. In this paper we discuss the analog results in bitopological settings.

Keywords: *ij*-semi-closure, *ij*-semi-interior, α-closure, bitopology.

Introduction

The study of bitopological spaces was first initiated by Kelly in 1963. A large number of papers have been done to generalize the topological concepts to bitopological setting. The closure and interior operators are the two important concepts in the study of general topological structures. These two operators are widely used in computer science applications especially in Rough set theory and in Fuzzy set theory. More over the closure and interior operators are very close to the upper and lower approximations in rough set theory respectively. The existence of two topologies on a non empty set X provides information about the set X in two different methods. The bitopological spaces have applications in data mining using rough sets. This article dealt with the closure and interior operators in bitopological settings. Andrijevic studied the relations among the operators' in general topological spaces. Maheshwari and Prasad [5], Jelic [3], Khedr et al. [4] and Sampath Kumar [6] studied the operator's namely semi-closure, pre-closure, semi-pre-closure and α–closure in bitopological settings. Andrijevic established the result $\alpha cl A = A \cup cl(int(cl A))$ for any subset A of a topological space (X, τ) where *cl*A and *int*A denotes the closure of A and interior of A respectively in (X, τ). He also established the similar results for other operators. In this paper we extend these results to bitopological spaces. In section 2 we recall the definitions of *ij*-operators in bitopological spaces and we study the relations among them in section 3. The symbol □ indicates the end of the proof. We use the notation *cl int cl*A for *cl*(*int*(*cl*A)).

Preliminaries

Throughout the paper the ordered triplet (X, τ_1, τ_2) is a bitopological space where τ_1 and τ_2 are topologies on X. Let A and B be subsets of the bitopological space (X, τ_1, τ_2).
The notations *j-cl*A and *j-int*A denote the closure of A and interior of A with respect to the topology τ_j respectively. X\A denotes the complement of A in X. The following definitions will be useful.

Definition 2.1. A is called

(i) *ij*-semi-open [5] in (X, τ_1, τ_2) if there exists a τ_i-open set U with $U \subseteq A \subseteq j\text{-}cl U$.

(ii) *ij*-pre-open [3] in (X, τ_1, τ_2) if there exists a τ_i-open set U with $A \subseteq U \subseteq j\text{-}cl A$.

(iii) *ij*-semi-pre-open or *ij*-β-open [4] in (X, τ_1, τ_2) if there exists an *ij*-pre-open set U in (X, τ_1, τ_2) with $U \subseteq A \subseteq j\text{-}cl U$ that is if $A \subseteq j\text{-}cl(i\text{-}int(j\text{-}cl A))$.

(iv) *ij*-α-open [3], [6] in (X, τ_1, τ_2) if $A \subseteq i\text{-}int(j\text{-}cl(i\text{-}int A))$.

The complement of an *ij*-semi-open set is *ij*-semi-closed. The *ij*-pre-closed sets, *ij*-semi-pre-closed sets and *ij*-α-closed sets will be analogously defined.

Maheshwari and Prasad [5] established the following theorem.

Theorem 2.2. The following results hold in a bitopological space.

(i) The union of an arbitrarily collection of *ij*-semi-open sets is *ij*-semi-open;

(ii) The intersection of two *ij*-semi-open sets is not *ij*-semi-open;

(iii) The intersection of an arbitrarily collection of *ij*-semi-closed sets is *ij*-semi-closed and

(iv) The union of two *ij*-semi-closed sets is not *ij*-semi-closed.

Jelic [3], Khedr et al. [4] and Sampath Kumar[6] respectively characterized *ij*-pre-open sets, *ij*-semi-pre-open sets and *ij*-α-open sets. The intersection of all *ij*-semi-closed sub sets of (X, τ_1, τ_2) containing a subset A of X is the *ij*-semi-closure of A, denoted by *ij-scl*A. The union of all *ij*-semi-open sets contained in A is called the *ij*-semi-interior of A, denoted by *ij-sint*A. The *ij*-pre-closure, *ij*-semi-pre-closure, *ij*-α-closure, *ij*-pre-interior, *ij*-semi-pre-interior and *ij*-α-interior will be analogously defined respectively denoted by *ij-pcl*A, *ij-spcl*A, *ij-*α*cl*A, *ij-pint*A, *ij-spint*A and *ij-*α*int*A.

Properties

Andrijevic [2] established the following results for any subset A of a general topological spaces.

(i) $scl A = A \cup int\, cl A$.

(ii) $sint A = A \cap cl\, int A$.

(iii) $\alpha cl A = A \cup cl\, int\, cl\, A$.

(iv) $\alpha int A = A \cap int\, cl\, int A$.

We extend the above results to bitopological spaces.

Proposition 3.1. For any subset A of (X, τ_1, τ_2), $A \cap j\text{-}cl(i\text{-}int A)$ is *ij*-semi-open and $A \cup j\text{-}int(i\text{-}cl A)$ is *ij*-semi-closed.

Proof. $j\text{-}cl(i\text{-}int(A \cap j\text{-}cl(i\text{-}int A))) = j\text{-}cl(i\text{-}int A \cap i\text{-}int(j\text{-}cl(i\text{-}int A)))$.

$= j\text{-}cl(i\text{-}int A)$.

$A \cap j\text{-}cl(i\text{-}int A) = A \cap j\text{-}cl(i\text{-}int(A \cap j\text{-}cl(i\text{-}int A)))$

$\subseteq j\text{-}cl(i\text{-}int(A \cap j\text{-}cl(i\text{-}int A)))$.

$A \cap j\text{-}cl(i\text{-}int A)$ is *ij*-semi-open.

This implies that $X \setminus (A \cup j\text{-}int(i\text{-}cl A)) = (X \setminus A) \cap j\text{-}cl(i\text{-}int(X \setminus A))$ is *ij*-semi-open. This proves that $A \cup j\text{-}int(i\text{-}cl A)$ is *ij*-semi-closed. □

Proposition 3.2. Let A be a subset of (X, τ_1, τ_2). Then $ij\text{-}sint\text{A} = \text{A} \cap j\text{-}cl(i\text{-}int\text{A})$.

Proof. Let B = ij-$sint$A = the union of all ij-semi-open sets contained in A. B is ij-semi-open and B $\subseteq$ A. Since B is ij-semi-open, $\text{B} \subseteq j\text{-}cl(i\text{-}int\text{B}) \subseteq j\text{-}cl(i\text{-}int\text{A})$. This implies that $\text{B} \subseteq \text{A} \cap j\text{-}cl(i\text{-}int\text{A})$. Now using Proposition 3.1, $\text{A} \cap j\text{-}cl(i\text{-}int\text{A})$ is ij-semi-open.

Since $\text{A} \cap j\text{-}cl(i\text{-}int\text{A}) \subseteq \text{A}$, by the definition of ij-$sint$A, $\text{A} \cap j\text{-}cl(i\text{-}int\text{A}) \subseteq \text{B}$ and that shows $ij\text{-}sint\text{A} = \text{A} \cap j\text{-}cl(i\text{-}int\text{A})$.

Proposition 3.3. Let A be a subset of (X, τ_1, τ_2). Then $ij\text{-}scl\text{A} = \text{A} \cup j\text{-}int(i\text{-}cl\text{A})$.

Proof. ij-sclA = intersection of all ij-semi-closed sets of X containing A

$= X \setminus ij\text{-}sint(X \setminus A)$,

$= X \setminus ((X \setminus A) \cap j\text{-}cl(i\text{-}int(X \setminus A))$, using Proposition 3.2

$= X \setminus ((X \setminus A) \cap (X \setminus j\text{-}int(i\text{-}cl\text{A}))$

$= \text{A} \cup j\text{-}int(i\text{-}cl\text{A})$. □

Proposition 3.4. $i\text{-}int(j\text{-}cl(i\text{-}int\text{A})) \subseteq ji\text{-}scl(ij\text{-}sint\text{A})$.

Proof. $ji\text{-}scl(ij\text{-}sint\text{A}) = ji\text{-}scl(\text{A} \cap j\text{-}cl(i\text{-}int\text{A}))$, using Proposition 3.2

$= (\text{A} \cap j\text{-}cl(i\text{-}int\text{A})) \cup i\text{-}int\,(j\text{-}cl(\text{A} \cap j\text{-}cl(i\text{-}int\text{A})))$, using Proposition 3.3

$\supseteq (\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))) \cup i\text{-}int(j\text{-}cl(i\text{-}int\text{A} \cap j\text{-}cl(i\text{-}int\text{A})))$

$= (\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))) \cup i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))$

$= i\text{-}int(j\text{-}cl\,(i\text{-}int\text{A}))$.

Lemma 3.5. For any sub set A of (X, τ_1, τ_2), $\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))$ is ij-α-open and $\text{A} \cup i\text{-}cl(j\text{-}int(i\text{-}cl\text{A}))$ is ij-α-closed.

Proof. $i\text{-}int(j\text{-}cl(i\text{-}int(\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))))) = i\text{-}int(j\text{-}cl(i\text{-}int\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))))$

$= i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))$. This implies that

$\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\,\text{A})) = \text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int(\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A})))))$

$\subseteq i\text{-}int(j\text{-}cl(i\text{-}int(\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A})))))$

$\text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))$is ij-α-open. This implies

$X \setminus (\text{A} \cup i\text{-}cl(j\text{-}int(i\text{-}cl\text{A}))) = (X \setminus A) \cap i\text{-}int(j\text{-}cl(i\text{-}int(X \setminus A)))$

= ij-α-open that further implies

$\text{A} \cup i\text{-}cl(j\text{-}int(i\text{-}cl\text{A}))$ is ij-α-closed. □

Proposition 3.6. $ij\text{-}\alpha int\text{A} = \text{A} \cap i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))$.

Proof. Let B = ij-αintA = the union of all ij-α-open sets contained in A.

Clearly B is ij-α-open and B $\subseteq$ A. Since B is ij-α-open,

$\text{B} \subseteq i\text{-}int(j\text{-}cl(i\text{-}int\text{B})) \subseteq i\text{-}int(j\text{-}cl(i\text{-}int\text{A}))$. This proves that

$B \subseteq A \cap i\text{-}int(j\text{-}cl(i\text{-}intA))$. Now using Lemma 3.5,

$A \cap i\text{-}int(j\text{-}cl(i\text{-}intA))$ is ij-α-open. By the definition of $ij\text{-}\alpha intA$,

$A \cap i\text{-}int(j\text{-}cl(i\text{-}intA)) \subseteq B$. Then it follows that $B = A \cap i\text{-}int(j\text{-}cl(i\text{-}intA))$.

Therefore $ij\text{-}\alpha intA = A \cap i\text{-}int(j\text{-}cl(i\text{-}intA))$. □

Proposition 3.7. $ij\text{-}\alpha clA = A \cup i\text{-}cl(j\text{-}int(i\text{-}clA))$.

Proof. $ij\text{-}\alpha clA$ = intersection of all ij-α-closed sub sets of X containing A

$= X \setminus ij\text{-}\alpha int(X\setminus A)$,

$= X \setminus((X\setminus A) \cap i\text{-}int(j\text{-}cl(i\text{-}int(X\setminus A))))$, using Proposition 3.6

$= X \setminus((X\setminus A) \cap (X\setminus i\text{-}cl(j\text{-}int(i\text{-}cl\, A))))$

$= A \cup i\text{-}cl(j\text{-}int(i\text{-}clA))$. □

Andrijevic[2] also established that $\alpha int(\alpha clA) = int\, clA$ and $\alpha cl(\alpha intA) = cl\ intA$ in unital topological spaces. However the corresponding results in bitopological spaces do not hold as shown below.

Proposition 3.8. $ij\text{-}\alpha int(ji\text{-}\alpha clA) \supseteq i\text{-}int(j\text{-}clA)$.

Proof. Using Proposition 3.6 and Proposition 3.7 we get

$ij\text{-}\alpha int(ji\text{-}\alpha clA) = ij\text{-}\alpha int(A \cup j\text{-}cl(i\text{-}int(j\text{-}clA)))$

$= (A \cup j\text{-}cl(i\text{-}int(j\text{-}clA))) \cap i\text{-}int(j\text{-}cl(i\text{-}int(A \cup j\text{-}cl(i\text{-}int(j\text{-}cl\, A)))))$

$\supseteq (A \cup j\text{-}cl(i\text{-}int(j\text{-}clA))) \cap i\text{-}int(j\text{-}cl(i\text{-}intA \cup i\text{-}int(j\text{-}cl(i\text{-}int(j\text{-}clA)))))$

$= (A \cup j\text{-}cl(i\text{-}int(j\text{-}clA))) \cap i\text{-}int(j\text{-}cl(i\text{-}intA \cup i\text{-}int(j\text{-}clA)))$

$= (A \cup j\text{-}cl(i\text{-}int(j\text{-}clA))) \cap i\text{-}int(j\text{-}cl(i\text{-}int(j\text{-}clA)))$

$= (A \cup j\text{-}cl(i\text{-}int(j\text{-}clA))) \cap i\text{-}int(j\text{-}clA)$

$= (A \cap i\text{-}int(j\text{-}clA)) \cup (j\text{-}cl(i\text{-}int(j\text{-}clA)) \cap i\text{-}int(j\text{-}clA))$

$= (A \cap i\text{-}int(j\text{-}clA)) \cup i\text{-}int(j\text{-}clA)$

$= i\text{-}int(j\text{-}clA)$. □

Proposition 3.9. $ij\text{-}\alpha cl(ji\text{-}\alpha intA) \subseteq i\text{-}cl(j\text{-}intA)$.

Proof. $X\setminus ij\text{-}\alpha cl(ji\text{-}\alpha intA) = ij\text{-}\alpha int(X\setminus(ji\text{-}\alpha intA))$,

$= ij\text{-}\alpha int(ji\text{-}\alpha cl(X\setminus A))$,

$\supseteq i\text{-}int(j\text{-}cl(X\setminus A))$, by Proposition 3.8

$= X\setminus i\text{-}cl(j\text{-}intA)$

This proves that $ij\text{-}\alpha cl(ji\text{-}\alpha int A) \subseteq i\text{-}cl(j\text{-}int A)$. □

Proposition 3.10. $ji\text{-}\alpha cl(ij\text{-}\alpha int(ji\text{-}\alpha cl A)) \supseteq j\text{-}cl(i\text{-}int(j\text{-}cl A))$.

Proof. $ji\text{-}\alpha cl(ij\text{-}\alpha int(ji\text{-}\alpha cl A)) \supseteq ji\text{-}\alpha cl(i\text{-}int(j\text{-}cl A))$, by Proposition 3.8

$= (i\text{-}int(j\text{-}cl A)) \cup j\text{-}cl(i\text{-}int(j\text{-}cl(i\text{-}int(j\text{-}cl A))))$, using Proposition 3.7

$= i\text{-}int(j\text{-}cl A) \cup j\text{-}cl(i\text{-}int(j\text{-}cl A))$

$= j\text{-}cl(i\text{-}int\,(j\text{-}cl A))$. □

Proposition 3.11. $ij\text{-}\alpha int(ji\text{-}\alpha cl(ij\text{-}\alpha int A)) \subseteq i\text{-}int(j\text{-}cl(i\text{-}int A))$.

Proof. $ij\text{-}\alpha int(ji\text{-}\alpha cl(ij\text{-}\alpha int A)) \subseteq ij\text{-}\alpha int(j\text{-}cl(i\text{-}int A))$, using Proposition 3.9

$= (j\text{-}cl(i\text{-}int A)) \cap i\text{-}int(j\text{-}cl(i\text{-}int(j\text{-}cl(i\text{-}int A))))$

$= j\text{-}cl(i\text{-}int A) \cap i\text{-}int(j\text{-}cl(i\text{-}int A))$

$= i\text{-}int(j\text{-}cl(i\text{-}int A))$. □

It is observed that $A \cap i\text{-}int(j\text{-}cl A)$ is not ij-pre-open as shown below.

Example 3.12. Let X = Z, the set of all integers. Let $\tau_1 = \{\emptyset, \{0, 2\}, Z\}$ and τ_2 be the digital topology[7]. Let A = {0, 1}. $2\text{-}cl A = \{0, 1, 2\}$ and $1\text{-}int\,(2\text{-}cl A) = \{0, 2\}$.
Let $B = A \cap 1\text{-}int(2\text{-}cl A) = \{0, 1\} \cap \{0, 2\} = \{0\}$. Since $1\text{-}int(2\text{-}cl B) = 1\text{-}int\{0\} = \emptyset$, B is not 12-pre-open. As $A \cap i\text{-}int(j\text{-}cl A)$ is not ij-pre-open, the result $ij\text{-}pint A = A \cap i\text{-}int(j\text{-}cl A)$
does not hold.
However the following proposition holds.

Proposition 3.13. $ij\text{-}pint A \subseteq A \cap i\text{-}int(j\text{-}cl A)$.

Proof. Let $B = ij\text{-}pint A$ = the union of all ij-pre-open sets contained in A

B is ij-pre-open and $B \subseteq A$. Since B is ij-pre-open, $B \subseteq i\text{-}int(j\text{-}cl B) \subseteq i\text{-}int(j\text{-}cl A)$. This proves that $B \subseteq A \cap i\text{-}int(j\text{-}cl A)$. □

Proposition 3.14. $ij\text{-}pcl A \supseteq A \cup i\text{-}cl(j\text{-}int A)$.

Proof.

$ij\text{-}pcl A = X \setminus ij\text{-}pint(X \setminus A)$,

$\supseteq X \setminus ((X \setminus A) \cap i\text{-}int(j\text{-}cl((X \setminus A))$, using Proposition 3.13

$= X \setminus ((X \setminus A) \cap (X \setminus i\text{-}cl(j\text{-}int A)))$

$= A \cup i\text{-}cl(j\text{-}int A)$. □

Proposition 3.15. If A is ij-semi-open then $A \cap i\text{-}int(j\text{-}cl A)$ is ij-pre-open.

Proof. $i\text{-}int(j\text{-}cl(A \cap i\text{-}int(j\text{-}cl A))) \supseteq i\text{-}int(j\text{-}cl(i\text{-}int A \cap i\text{-}int(j\text{-}cl A)))$

$= i\text{-}int(j\text{-}cl(i\text{-}int A))$.

Since A is ij-semi-open, $j\text{-}cl A = j\text{-}cl(i\text{-}int A)$ so that

i-int(j-clA) = i-int(j-cl(i-intA)). Therefore

$A \cap i\text{-}int(j\text{-}clA) = A \cap i\text{-}int(j\text{-}cl(i\text{-}intA)) \subseteq i\text{-}int(j\text{-}cl(i\text{-}intA))$

$\subseteq i\text{-}int(j\text{-}cl(A \cap i\text{-}int(j\text{-}clA)))$.

$A \cap i\text{-}int(j\text{-}clA)$ is ij-pre-open. □

Corollary 3.16. If A is ij-semi-closed then $A \cup i\text{-}cl(j\text{-}intA)$ is ij-pre-closed.

Proof. Suppose A is ij-semi-closed. Then X\A is ij-semi-open. Therefore, using Proposition 3.15, $(X\setminus A) \cap i\text{-}int(j\text{-}cl(X\setminus A))$ is ij-pre-open that implies $X\setminus(A \cup i\text{-}cl(j\text{-}intA))$ is ij-pre-open. This proves that $A \cup i\text{-}cl(j\text{-}intA)$ is ij-pre-closed. □

It is observed that $A \cap j\text{-}cl(i\text{-}int(j\text{-}clA))$ is not ij-semi-pre-open as shown below.

Example 3.17. Let X = Z, the set of all integers. Let $\tau_1 = \{\emptyset, \{0, 2\}, Z\}$ and τ_2 be the digital topology[7]. Let A = {1, 2}. 2-clA = {0, 1, 2} and 2-cl(1-int(2-clA)) = 2-cl{0, 2} = {0, 2}. Let B = A $\cap$ 2-cl(1-int (2-clA)) = {1, 2} $\cap$ {0, 2} = {2}.

Since 2-cl(1-int(2-clB)) = 2-cl(1-int{2}) = 2-$cl\emptyset = \emptyset$, B is not 12-semi-pre-open.

Proposition 3.18. $ij\text{-}spintA \subseteq A \cap j\text{-}cl(i\text{-}int(j\text{-}clA))$.

Proof. Let B = ij-$spint$A

= the union of all ij-semi-pre-open sets of X contained in A.

B is ij-semi-pre-open and $B \subseteq A$. Since B is ij-semi-pre-open,

$B \subseteq j\text{-}cl(i\text{-}int(j\text{-}clB)) \subseteq j\text{-}cl(i\text{-}int(j\text{-}clA))$. □

Proposition 3.19. $ij\text{-}spclA \supseteq A \cup j\text{-}int(i\text{-}cl(j\text{-}intA))$.

Proof. $ij\text{-}spclA = X\setminus ij\text{-}spint(X\setminus A)$,

$\supseteq X\setminus ((X\setminus A) \cap j\text{-}cl(i\text{-}int(j\text{-}cl(X\setminus A))))$, using Proposition 3.18

$= X\setminus ((X\setminus A) \cap (X\setminus j\text{-}int(i\text{-}cl(j\text{-}intA)))$

$= A \cup j\text{-}int(i\text{-}cl(j\text{-}intA))$. □

Proposition 3.20. If A is ji-semi-closed then $A \cap j\text{-}cl(i\text{-}int(j\text{-}clA))$ is ij-semi-pre-open.

Proof . Let $B = A \cap j\text{-}cl(i\text{-}int(j\text{-}clA))$.

$j\text{-}cl(i\text{-}int(j\text{-}clB)) \supseteq j\text{-}cl(i\text{-}int(j\text{-}cl(i\text{-}intA \cap j\text{-}cl(i\text{-}int(j\text{-}clA))))$

$= j\text{-}cl(i\text{-}int(j\text{-}cl(i\text{-}intA)))$

$= j\text{-}cl(i\text{-}intA))$. (*)

Since A is ji-semi-closed, $i\text{-}intA = i\text{-}int(j\text{-}clA)$ so that

$j\text{-}cl(i\text{-}intA)) = j\text{-}cl(i\text{-}int(j\text{-}clA))$. Therefore $B = A \cap j\text{-}cl\,(i\text{-}int(j\text{-}clA))$

$= A \cap (j\text{-}cl(i\text{-}intA)) \subseteq j\text{-}cl(i\text{-}intA)$

$\subseteq j\text{-}cl\,(i\text{-}int(j\text{-}clB))$ by (*)

This proves that $A \cap j\text{-}cl(i\text{-}int(j\text{-}clA)) = B$ is ij -semi-pre-open. □

Corollary 3.21. If A is *ji*-semi-open then A $\cup$ *j-int*(*i-cl*(*j-int*A)) is *ij*-semi-pre-closed.

Proof. Suppose A is *ji*-semi-open. Then X\A is *ji*-semi-closed.

So, using Proposition 3.20, (X\A) $\cap$ *j-cl* (*i-int*(*j-cl*(X\A))) is *ij*-semi-pre-open that implies X\ (A $\cup$ *j-int*(*i-cl*(*j-int*A))) is *ij*-semi-pre-open.

This proves that A $\cup$ *j-int* (*i-cl*(*j-int* A)) is *ij*-semi-pre-closed.

References

[1] Andrijevic D (1984) Some properties of the topology of α-sets. *Mat. Vesnik* 36: 1-10.

[2] Andrijevic D (1986) Semi-pre-open sets. *Mat. Vesnik* 38: 24-32.

[3] Jelic M (1990) A decomposition of pairwise continuity. *J.Inst. Math. Comput. Sci. Math.Ser.*3: 25-29.

[4] Khedr FH, SM Al Areefi and T Noiri (1992) Precontinuity and semi-precontinuity in bitopological spaces. *Indian J. pure appl. Math.* 23(9): 625-633.

[5] Maheshwari SN and R. Prasad (1977/78) Semi-open sets and semi-continuous functions in bitopological spaces. *Math. Notae* 26: 29-37.

[6] Sampath Kumar S (1997) On a decomposition of pairwise continuity. *Bull. Cal. Math. Soc.* 89: 441-446.

[7] Thangavelu P (2007) On the subspace topologies of the Khalimsky topology, *Proc.Second International Conference on Mathematics: Trends and Developments, The Egyptian Mathematical Society, Cairo, 27-30, Dec.2007 Vol (III) Top.& Geom.*: 157-168.