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Precalculus Algebra: Original Purpose – Its Relevancy to Students' Needs – and Some Suggestions

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Abstract

Our purpose of this paper is to discuss who should take Precalculus Algebra, not the content of the course. Precalculus Algebra was originally intended for students who would progress to Calculus. However, many non-technical degree programs now require their majors to take it, even though it may be a terminal math course for these students. A review of the mathematics curriculum at various universities was conducted in order to identify similarities and differences regarding this particular course. In addition, a student survey was conducted at Alabama A&M University (AAMU) to determine students' majors, their motivation for taking Precalculus Algebra, their attitude toward it, and the number of times they had taken it, reflecting their long-term success or failure with the course and its concepts. The curriculum reviews and the student surveys led us to question whether we, in the education community, are providing math skills that are relevant to the future careers of the students who are taking this as a terminal course. We propose alternative solutions and content for the different tracks of students taking Precalculus Algebra.

Keywords: Precalculus Algebra, Calculus, Student Needs, Relevancy to Mathematics curricula

Introduction

Precalculus Algebra has been discussed for many years with respect to the initial rationale for the course, the content of the course, the processes that place students in the course, the ultimate success of these students in the course, and the changes that departments have decided to make to the course in order to better accomplish departmental objectives (Small, 2002) (Harrell, 2003) (Bressoud, 2007) .

Precalculus (= Pre-Calculus) means 'before calculus' and it is indicative of the undergraduate courses that prepare students for calculus. In some colleges, precalculus may be College Algebra and elsewhere precalculus is a combination of both College Algebra and Trigonometry. Regardless of the format of precalculus, its sole purpose is to prepare students for a calculus course. Calculus started its journey about 300 years ago and over the years it has contributed significantly to the advancement of science, engineering and technology. To study almost any area of science and engineering, calculus is essential. The content of the Precalculus Algebra covers the algebra of polynomial, rational, exponential and logarithmic functions, algebraic equations, linear and quadratic equations, linear and quadratic inequalities, systems of equations and inequalities, and the binomial theorem. Additional topics may include matrices, Cramer's rule, mathematical induction and partial fractions.

Although the initial goal of Precalculus Algebra was to prepare students for Calculus, more college students study precalculus than calculus (Steen, 2006). Only 10-15% (Herriott, 2006) of students who take precalculus courses ever go on to take calculus (Dunbar, 2006). We have found from our personal experience that many students who take precalculus are not well-prepared for calculus and never complete it. As a result of this, students lose the opportunity to pursue math or math-based disciplines. For too many students, precalculus is the end of their study and interest in mathematics (Steen, 2006). The course has been used as a filter when it should be used as a pump, and it demoralizes students (Herriott, 2006). A significant number of students take the course because it is a requirement of their degree program. These students do not go on to take Calculus; and, in fact, Precalculus Algebra becomes a terminal mathematics course for them. Complicating the situation is the fact that many of these students are underprepared for Precalculus Algebra. When they

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come to the course, the lectures may sound like a foreign language (Small, 2006). Although a majority of these students start with a developmental math course, they still have problems in Precalculus Algebra.

In addition to our precalculus algebra students' fear of mathematics, they also have problems with the vocabulary we use when we teach the course and the vocabulary we use in the instructions for working problems. Frequently students will come to us and say, "When you show me how to do this, I know how; I just don't know when." This statement tends to make us believe that their approach to mathematics is simply one of mechanics rather than one of understanding the processes. This is especially true of students in non-technical fields.

This fear also frequently causes students to delay taking Precalculus Algebra possibly even until their last semester prior to graduation, and it is not uncommon for students to take the course a number of times. The fear, delay, and numerous failing grades have a significant impact on these students. Graduation can be delayed or even lost. More importantly, it negatively impacts their self-esteem. Finally, because students do not see the connection between the course and their major, it is not easy to motivate them to take initiative in the course.

Data analysis is a major activity in our modern society and even ordinary citizens need to deal with numbers and data. No matter where one looks (e.g. media, government, weather forecasts, investment, medical report, economic growth/decline, climate change etc.), one has to analyze charts and graphs. Our society needs quantitatively literate people (Steen, 2003), (Steen, 2004), (National Research Council, 2003a), (National Research Council, 2003b). It has been suggested that underprepared students should be steered in other quantitative directions (Steen 2006).

Students in non-technical majors are often weak in mathematics and fearful of it, yet they earn excellent grades in the courses specific to their major. Precalculus Algebra continues to be a standard course in the curriculum, including a General Education requirement, of many programs at a number of colleges and universities. Texas and Louisiana have legislated that a student must pass a math course at or above the level of College Algebra before they are permitted to enter their third year of college (Small, 2006). This raises questions about what the focus of Precalculus Algebra should be. In the March 2008 issue of the MAA Focus, the association announced a call for articles that would address the issues of the students who take Precalculus Algebra and the benefits the course would provide for these students (MAA, 2008). Even though the MAA Notes volume has been tabled for now, we will present the findings of a survey we conducted in the spring semester of 2008.

Approach of Alabama A&M University (Alabama A&M University, 2008)

Alabama A&M University (AAMU) offers four courses below the Calculus level for degree credit. The course Modern Mathematics covers the metric system, sets, base numeration systems, systems of whole numbers, systems of integers, elementary number theory, elementary logic, relations, and functions. The course Finite Mathematics covers sets, counting, permutations, combinations, basic probability theory (including Baye's Theorem), statistical concepts (including binomial distributions and normal distribution), matrices and their applications to Markov chains and decision theory. Additional topics that may be included are symbolic logic, linear models, linear programming, the simplex method and applications. The other two offered courses are Precalculus Algebra and Precalculus Trigonometry. However, not all of the courses are available for credit in every program.

Some programs offer a choice between Finite Mathematics and Precalculus Algebra. Students in the School of Education have varying requirements, depending upon the area of specialization, but Precalculus Algebra is included in all of the Education programs. Programs in the School of Business require that their students successfully complete Precalculus Algebra and Calculus and Its Applications (often called Business Calculus). Students majoring in Biology are required to take Precalculus Algebra, Precalculus Trigonometry, Calculus I and Calculus II. Calculus I is the first course for which students in Science, Technology, Engineering and Mathematics (STEM) disciplines can receive program credit. In addition, AAMU offers two courses, Intermediate Algebra and Precalculus Algebra and Trigonometry, which are developmental courses for students in the STEM disciplines.

AAMU Survey

The Department of Mathematics at AAMU conducted a survey in all sections of Precalculus Algebra (MTH 112) in the spring semester of 2008. The survey consisted of ten questions that addressed a number of topics as suggested by the MAA Focus call for articles. The survey was free response; not every student answered every question. We focused on four questions and the corresponding responses are presented below.

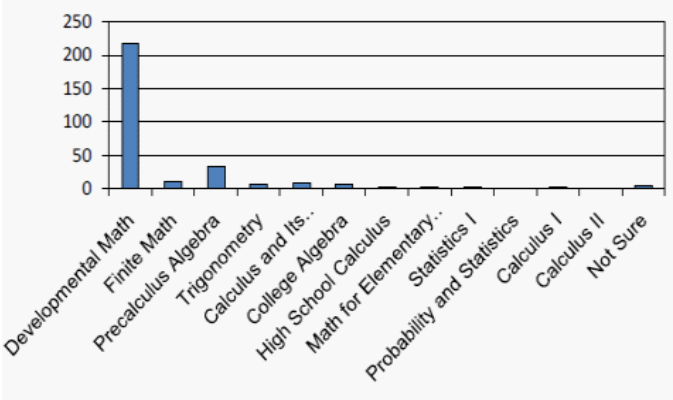

Fig. 1. The Last Math Course Taken.

Question #1. What was the last math course you took?

73.1% of our students took a developmental math course prior to taking Precalculus Algebra. Students are preparing themselves for the education they need prior to taking Precalculus Algebra. Approximately 12% did not pass and are retaking the course.

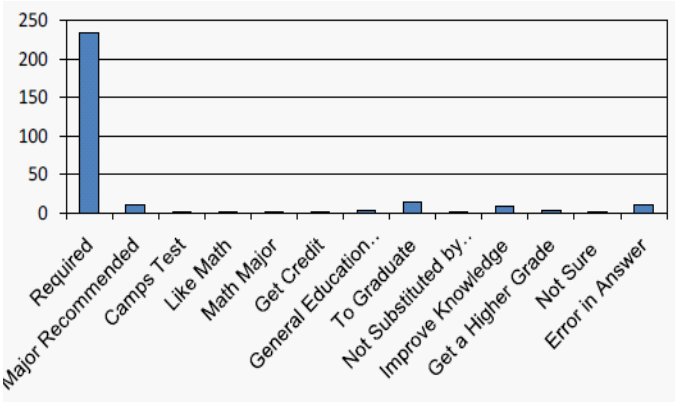

Fig. 2. The Reasons for Enrollment in MTH 112.

Question # 2. Why are you enrolled in this MTH 112 course now?

About 81% of the students took Precalculus Algebra because it was a degree requirement. Very few students took the course because of their interest in math, as shown by the counts in "Like Math" and "Improve Knowledge". It appeared that 67.3% of the students who are required to take precalculus are not required to take Calculus; only 32.7% of the students need Calculus for their major.

Question #3. What math courses are you planning on taking next (if any)?

The majors of about 30% of the students surveyed do not require any math course for their graduation. Only about 16% intend to take either Calculus I or Calculus II. About 10% of the students are not sure whether they have to take a math course after MTH 112, and it is because these students have not yet decided their major. About 20% of the students are planning to take Calculus and Its Applications (Business Calculus).

Question #4. Is this the first time you have taken MTH 112? If not, how many times have you taken it previously to this semester?

Significantly more students responded to this question than any other. 59.5% of the students were taking the course for the first time; 37.6% had taken the course more than once. This is not unusual in our experience at AAMU.

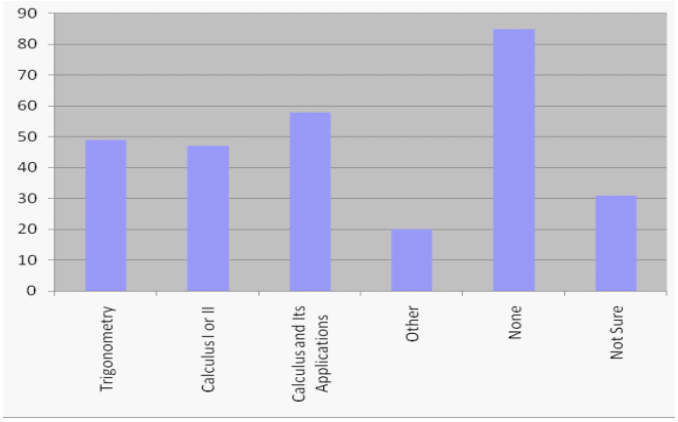

Fig. 3. The Plan of the Next Math Course.

The survey results raise the following questions:

- Are we doing what is best for our students by requiring Precalculus Algebra when Calculus is not required?
- Do we need a Precalculus Algebra for students who will take Calculus and Its Applications and a different one for students in STEM disciplines?
- Why are we so rigid with respect to this issue?

Approach of Other Selected Colleges and Universities: Who Is Doing What?

In the process of developing this paper and thinking about the MAA question and issues, we began to ask how the issue is being addressed at universities across the country. In addition to Alabama A&M University, which is where we teach, we checked state universities in Oklahoma, Texas, and Tennessee as well as private universities in Georgia and Alabama.
In order to obtain an idea of policies at other universities, we reviewed course descriptions in states other than Alabama as well as in private schools.

University of Oklahoma (University of Oklahoma, 2010)

At the University of Olahoma, students are required to take one course in mathematics for their General Education requirement. Three math courses are available for review, not for credit toward graduation.

There are four math courses that are offered for credit prior to Calculus. 1) *Mathematics for Critical Thinking* is used for the critical evaluation of quantitative information and arguments that include logic and critical appraisal of graphs and tables. This course also deals with simple mathematical models and elementary statistics. 2) *Introduction to Elementary Functions* deals with basic algebraic skills such as multiplying and factoring polynomials, rational expressions, linear equations and inequalities, exponents and radicals, and absolute values. It also covers the concept, notation, algebra of functions, (linear, polynomial, rational, exponential, and logarithmic) and systems of equations. 3) *Elementary Functions* includes the properties of exponential and logarithmic functions, trigonometric functions and their inverses, trigonometric identities and equations, conic sections, polar coordinates, Demoivre's theorem, discrete algebra, induction, limits and continuity. 4) *Precalculus for Business, Life and Social Sciences* covers linear functions, exponential and logarithmic functions, systems of linear equations and inequalities, matrices and operations on them, linear programming, introductory trigonometry, and elementary probability and statistics.

University of Texas – Austin (University of Texas - Austin, 2010)

The University of Texas, Austin offers *Introduction to Mathematics* to satisfy the General Education requirement in mathematics. It is a terminal course that covers number theory (which includes divisibility, prime numbers, the Fundamental Theorem of Arithmetic, GCD, Euclidean Algorithm, modular arithmetic,

special divisibility tests), probability (which includes definition, laws, permutations and combinations), network theory (covering Euler circuits, traveling salesman problem, bin packing), and game theory.

The University offers for the non-technical major *Applicable Mathematics:* this course allows students to apply mathematics to a variety of problems. The course is designed for students who are lacking mathematical skills. It includes linear and quadratic equations, systems of linear equations, matrices, probability, statistics, exponential and logarithmic functions, and mathematics of finance.

The University offers *Elementary Functions* and *Coordinate Geometry* for students who must take *Calculus*. The course covers sets, algebra of functions, inverse functions, logarithmic functions, exponential functions, trigonometric functions, inverse trigonometric functions, polynomials, and the range, domain and graphs of these functions.

University of Tennessee – Knoxville (University of Tennessee - Knoxville, 2010)

The University of Tennessee, Knoxville offers *Intermediate Algebra*, *Algebra Workshop*, and *College Algebra*. The first course does not count toward graduation. The second course is a self-paced tutorial for students who need additional help. The last course is a review of algebraic functions, equations, and inequalities for students who are majoring in a field other than the physical sciences, engineering, mathematics, or computer science. This course is designed as a prerequisite for courses that cover exponential and logarithmic functions, interest and annuities, linear systems and matrices, optimization, and business calculus.

The University also offers *Precalculus I* as a review of algebraic, logarithmic, exponential, and trigonometric functions for students in science, engineering, mathematics and computer science. *Finite Mathematics* is another course offered by the University and it is designed to explore the mathematics of finance and to apply systems of linear equations and inequalities to realistic application problems. The purpose of this course is to provide the student with mathematical methods for solving a variety of problems such as investment analysis, profit and loss analysis, production scheduling, and resource allocations. Students who take the course are usually majoring in business, economics, social science, agriculture, architecture, communications, or human ecology. The course covers linear, rational, exponential and logarithmic functions, mathematical modeling, simple and compound interest, annuities, amortization, systems of linear equations and inequalities, matrices, game theory, Leontief-Output analysis, linear programming, the Simplex method, and duality for the Simplex method.

Oakwood University (Oakwood University, 2010)

Oakwood University offers *Fundamental Concepts of Mathematics*, and it focuses on critical thinking skills, sets, number theory, the real number system, algebra, graphs, consumer mathematics, and geometry. The University also offers *Introductory College Algebra* and reviews the fundamental ideas of algebra. The course covers linear and quadratic equations and inequalities, rational expressions and functions, polynomials, factoring, systems of equations and graphs. These courses are neither for mathematics majors nor mathematics minors. The University also offers three other math courses preparing students for calculus: *Precalculus Algebra*, *Precalculus Trigonometry*, and *Precalculus Algebra & Trigonometry* for science majors, each of which covers the standard course content.

Observations and Perspectives

All of the schools discussed have General Education requirements, but due to the difference in degree programs and technical studies, these requirements differ by school. In addition, College Algebra courses that are not like Precalculus in content are often offered but not for credit.

At The University of Oklahoma only STEM majors take College Algebra and business majors take another course that includes Trigonometry. Students in psychology, sociology, philosophy and political science take a course involving logic.

At The University of Texas Austin, Precalculus Algebra is taken by those who need Calculus. The course is rarely offered and is taken by those not prepared for higher math courses.

At Oakwood University, Precalculus Algebra is only required for physical science majors. Life Science and Business majors are only required to take Fundamental Concepts of Mathematics.

Proposed solution

We doubt that AAMU is the only university with non-Calculus programs requiring Precalculus. As shown by responses to our survey in addition to our experiences in class, students, especially those who do not need Calculus, do not connect Precalculus to their future careers. Would it not be more logical to teach topics the students will apply? Thus, we agree with MAA – there needs to be dialog and change. Using approaches from other schools as reference, we get some ideas for our programs. We propose two tracks for students: a Calculus track and a non-Calculus track.

STEM-discipline Calculus Track

A fundamental belief is that Precalculus should help with Calculus. We believe that the changes we propose will accomplish that. Certainly, much of the current course content, such as the algebra of rational, exponential, and logarithmic functions, is necessary for Calculus. The Rational Zero Theorem can also be helpful in Calculus, especially in the area of curve sketching. Although Calculus is limited to the set of Real Numbers, students still need a basic understanding of complex numbers. The Binomial Theorem is a must, after all, how often do we fight the simple battle $(a + b)^2 = a^2 + b^2$?

Although factoring techniques are taught in prior algebra courses, students still come into Precalculus Algebra weak in the area. Adding, subtracting, multiplying and simplifying rational expressions are often difficult concepts for students, especially when determining least common denominators and greatest common divisors. Therefore, spending more time and practice on the techniques would be highly advantageous to students as they

proceed into Calculus, where simplifying derivatives is often a frustrating and difficult portion of the course for them.

In addition, we propose more emphasis on partial fractions since they are instrumental as an integration technique and in Laplace transforms in solving differential and integral equations. Finding zeros of polynomials is an interesting topic that becomes useful when we teach curve sketching in Calculus. Using critical numbers and inflection points, we use zeros in order to produce a more accurate graph.

In today's world there are majors, such as Biology, which have not been traditionally focused on the need for mathematics. However, the combination of math and biology has created ever-expanding fields and needs. The mathematical models and theorems are being applied to many areas areas such as those of inherited diseases, fuel regeneration, and insects and their interaction with other insects. As mathematics educators, we need to help our students understand how our field ties to these other areas.

The time spent on linear and quadratic functions can be shortened since they are covered in developmental courses. Upper and lower bounds for polynomials, zeros, and Descartes Rule of Signs are important topics but are quite technical for an introductory math course.

Non-Calculus Track

For students who are not on the Calculus track, we propose a College Algebra that teaches basic algebra and an understanding of other mathematical concepts that are necessary in daily life. For example, even though numerous students felt they would never need graphs or have to understand graphing techniques, the reality is that graphs are all around us – in newspapers and magazines. The course would spend more time on factoring, solving linear and quadratic equations, elementary statistics, and the binomial theorem as applicable to second and third degree equations, which are easily factorable. Since these students do not need Trigonometry, it would not be a part of the new course. We also propose introducing and teaching basic statistics in this course. After all, statistical information is reported to us on television and in print. If people do not have an understanding of the basic calculations and vocabulary, such as mean, mode and median, how will they know how to correctly interpret and question the numbers? Basic logic would also be addressed in this course to develop more critical thinking.

Conclusion

In summary, we have presented our survey and data, consisting of four questions and results. A large percentage of students take a developmental course before taking Precalculus Algebra. They take the course because it is required by their degree program, even though they do not have to take any more Math, or are not sure if they must. Over half of the students were taking the course for the first time.

Few students who take precalculus go on to take calculus, thus ending their mathematics studies, and when they are unsuccessful at completing the course, it negatively impacts their self-esteem. More importantly, they do not receive the quantitative education that would enable them to analyze data and graphs, make statistical calculations, or apply other areas of mathematics in their careers. We believe there should be a Calculus track and a non-Calculus track for students, we have presented the advantages of such an approach and the content for each track.

References

Alabama A&M University (2008) http://www.aamu.edu/acadaffairs/docs/BULLETIN_2008-2011.pdf

Bressoud DM (2007) Launchings from the CUPM Curriculum Guide: Return to College Algebra.Retrieved December, 2008, from http://www.maa.org/columns/launchings/launchings_03_07.html

Dunbar S (2006) Enrollment Flow to and From Courses below Calculus, in A Fresh Start for Collegiate Mathematics, MAA Notes #69.

Harrell R & T Lakins (2003) Assessing Introductory Calculus and Precalculus Courses. Retrieved December, 2008, from http://www.maa.org/SAUM/cases/harrell-lakins1105-saum.pdf

Herriott SR (2006) Changes in College Algebra, in A Fresh Start for Collegiate Mathematics, MAA Notes #69.

Mathematical Association of America (2008). College Algebra and Precalculus: A Call for Papers. MAA Focus, 28(3), 19.LRe

National Research Council (2003a) Evaluating and Improving Undergraduate Teaching in Science, Technology, Engineering, and Mathematics. Washington, DC: National Academy Press.

National Research Council (2003b) Improving Undergraduate Instruction in Science, Technology, Engineering, and Mathematics: Report of a Workshop. Washington, DC: National Academy Press.

Oakwood University (2010) http://www.oakwood.edu/Math_Computer_Sciences/default.aspx?id=20

Small D (2002) An Urgent Call to Improve Traditional College Algebra Programs. Retrieved December, 2008, from http://www.maa.org/t_and_l/urgent_call.html

Small D (2006) College Algebra: A Course in Crisis in A Fresh Start for Collegiate Mathematics, MAA Notes #69,.

Steen LA and BL Madison (2003) Quantitative Literacy: Why Numeracy Matters for Schools and Colleges, Princeton, NJ, National Council on Education and the Disciplines.

Steen LA (2004) Achieving Quantitative Literacy, Washington, DC: The Mathematical Association of America, MAA Notes No. 62.

Steen LA (2006) Twenty Questions about Precalculus, in A Fresh Start for Collegiate Mathematics, MAA Notes #69.

University of Oklahoma (2010) http://catalog.ou.edu/

University of Tennessee, Knoxville (2010) http://diglib.lib.utk.edu/dlc/catalog/u2007.htm

University of Texas, Austin (2010) http://registrar.utexas.edu/catalogs/.